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Effects of Temperature on the Existence of Resource Based Predator-Prey System: Modelling and Stability Analysis

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Abstract

Thermal pollutants include the waste heat chiefly from atomic, nuclear and thermal power plants, which adversely affect the aquatic environment. Coal fired or nuclear fuel-fired steam power plants are associated with the problem of thermal pollution. The study of the effects of temperature on the existence of resource based predator-prey system is carried out in the paper using mathematical model. The study has been conducted considering prey-predator system in which the parameters like growth rate, death rate, carrying capacity and interaction coefficients are assumed to be the functions of temperature which itself is changing with time. In writing the model, a separate dynamical equation for the temperature is supplemented with the equations for prey-predator system. In this paper, the effect of temperature is assumed to occur due to the natural reasons such as seasonal or climatic changes and also as a result of the hot water discharge from the industries. The mathematical model to study the effects of temperature on the existence of resource based prey-predator species system is analyzed using stability theory for two feasible equilibrium points. It is found that the first positive equilibrium state which is unstable without diffusion may become stable by considering diffusion and advection. On the basis of this analysis, conditions for survival or extinction of the prey and predator populations have been derived.

Keywords: Mathematical model, thermal pollution, diffusion, advection, stability, prey and predator.

Introduction

In nature, the dynamics of the population is governed by the environmental factors such as temperature, precipitation etc. Temperature is one of the important environmental factors and its variation is very important ecologically. It has been observed in nature that the growth rate, carrying capacity and interaction coefficient depend on the variations of temperature in the habitat. Thermal pollutants include the waste heat from atomic, nuclear and thermal power plants, which adversely affect the aquatic environment. Coal fired or nuclear fuel-fired steam power plants are associated with the problem of thermal pollution. A plant operating at 40% efficiency generates 16.7 joules of waste heat for every 41.8 jouls of fuel burnt. The condenser coils are cooled with water from nearby river or lake and discharged back to the later with its temperature raised by about 10°C in the process. This has obviously harmful effect on the aquatic life. It decreases DO (dissolved oxygen) of water and adversely affects aquatic life. The thermal pollution of water creates two major problems (i) The activity of biological life is more at higher temperature and hence as temperature of water rises, there is more demand of dissolved oxygen and (ii) as the temperature of water rises, the amount of dissolved oxygen in water decreases. Hence at higher temperature, less amount of dissolved oxygen will be present in water and it may be fatal for aquatic life. The hottest water kills some animals outright and the less hot water may also affect the fauna. In this paper, therefore, a mathematical model has been proposed to study the effect of temperature variations due to the hot water discharge from industries on the dynamics of predator-prey systems.

The study of the effects of temperature on the existence of resource based predator-prey system is carried out in the paper using mathematical model. The study has been conducted considering prey-predator system in which the parameters like growth rate, death rate, carrying capacity and interaction coefficients are assumed to be the functions of temperature which itself is changing with time. The growth rate and carrying capacity of prey population is considered as a decreasing function of temperature and the death rate of predator to be an increasing function of the temperature. The

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predation rate is considered to be a decreasing function of temperature. The growth rate of prey species is assumed to be an increasing function of aquatic plant biomass. In the mathematical model, a separate dynamical dynamical equation for the temperature is supplemented with the equations for prey-predator system. The mathematical model to study the effects of temperature on the existence of resource based prey-predator species system is analyzed using stability theory. In this paper, the effect of temperature is assumed to occur due to the natural reasons such as seasonal or climatic changes and also as a result of the hot water discharge from the industries.

Mathematical Model

The mathematical model is given by the following system of non linear partial differential equations in a linear habitat $0 \le x \le a$,

$$\frac{\partial H}{\partial t} = r_1(N,T)H - \frac{s_0 H^2}{k_1(T)} - a(T)PH - u_1\frac{\partial H}{\partial x} + D_1\frac{\partial^2 H}{\partial x^2} \tag{2.1}$$

$$\frac{\partial P}{\partial t} = -r_2(T)P + \alpha_1 a(T)PH - \gamma P^2 - u_2\frac{\partial P}{\partial x} + D_2\frac{\partial^2 P}{\partial x^2} \tag{2.2}$$

$$\frac{\partial N}{\partial t} = \mu N - mN^2 - \alpha_2 NH - u_3\frac{\partial N}{\partial x} + D_3\frac{\partial^2 N}{\partial x^2} \tag{2.3}$$

$$\frac{\partial T}{\partial t} = Q_0 - \alpha(T - T_0) + D_4\frac{\partial^2 T}{\partial x^2} \tag{2.4}$$

With the initial conditions which are given as follows:

$H(x,0) = f_1(x) \ge 0,\ P(x,0) = f_2(x) \ge 0,$

$N(x,0) = f_3(x) \ge 0, T(z,0) = f_4(x) \ge 0$

The model is associated with the following boundary conditions:

$$H = H^*,\ \ P = P^*,\ \ N = N^*,\ \ T = T^* \quad at \quad x = 0, a.$$

The model may be associated with the no flux boundary conditions also. For the analysis, we consider-

$$r_1(N,T) = r_{10} + r_{11}N - r_{22}T \quad , \quad k_1(T) = k_{10} - k_{11}T$$

$$r_2(T) = r_{20} + r_{22}T \quad , \quad a(T) = a_0 - a_{11}T;$$

Where H=Prey density, P = Predator density, N = Plant biomass(resourceforherbivores),T=Temperature, $r_1(N,T)$ =Intrinsic Growth rate of prey population, $r_2(T) =$ Death rate of predator population,$k_1(T)$ =Carrying capacity of H or maximum population that the environment can sustain,s_0 =Positive parameter which measures the population response to to the environmental stress,$a(T)$ =Predation rate,$\gamma =$ Intraspecific interaction coefficient of predator population (crowding effect), $\mu =$ Natural growth rate of N, $m =$ Depletion rate of N because of intraspecific competition (crowding effect),α_1 =Biomass conversion coefficient $(0 < \alpha_1 < 1)$,$\alpha_2 =$ Depletion rate of N because of consumption by prey population,u_i=Advection (Convection) coefficient, where i=1,2,3D_i=Diffusion (dispersal) coefficient, where i=1,2,3,4α =Constant, usually referred as the coefficient of surface heat transfer,$T_0 =$ Average temperature ofthe environment,Q_0 =Input rate of hot water from industries.And

$r_{10,}r_{20}, r_{11}, r_{22}, a_0, a_{11}, k_{10}, k_{11}, \gamma, \mu, m, \alpha_1, \alpha_2$ *and* a are all positive constants.

Mathematical Model Without Diffusion And Advection

In case of no diffusion the mathematical model is given by the following system of ordinary differential equations:

$$\frac{dH}{dt} = r_1(N,T)H - \frac{s_0 H^2}{k_1(T)} - a(T)PH \tag{3.1}$$

$$\frac{dP}{dt} = -r_2(T)P + \alpha_1 a(T)PH - \gamma P^2 \tag{3.2}$$

$$\frac{dN}{dt} = \mu N - mN^2 - \alpha_2 NH \tag{3.3}$$

$$\frac{dT}{dt} = Q_0 \ - \alpha(T - T_0) \tag{3.4}$$

Uniform Equilibrium Points

The uniform equilibrium points of the model are obtained as follows:

The **first equilibrium point** $E_1(\overline{H}, \bar{P}, \bar{N}, \bar{T})$ is given by-

Where, $\bar{P} = 0$, $\bar{H} = \frac{r_{10} + r_{11}\frac{\mu}{m} - r_{22}\bar{T}}{r_{11}\frac{\alpha_2}{m} + \frac{s_0}{k_1(\bar{T})}} > 0$, provided

$$r_{10} + r_{11}\frac{\mu}{m} > r_{22}\bar{T}$$

$\bar{N} = \frac{\mu - \alpha_2\bar{H}}{m} > 0\ provided\ \mu > \alpha_2\bar{H}$, $\bar{T} = \frac{Q_0 + \alpha T_0}{\alpha}$.

The **second equilibrium point** $E_2(\tilde{H}, \tilde{P}, \tilde{N}, \tilde{T})$ is given by- Here $\bar{T} = \tilde{T}$, $\bar{N} = \tilde{N}$, $\tilde{H} = \frac{r_2(\tilde{T}) + \gamma\tilde{P}}{\alpha_1 a(\tilde{T})} > 0$ and

$\tilde{P} = \frac{r_{10} + r_{11}\frac{\mu}{m} - r_{22}\tilde{T} - A\frac{r_2(\tilde{T})}{\alpha_1 a(\tilde{T})}}{\frac{\gamma A}{\alpha_1 a(\tilde{T})} + a(\tilde{T})} > 0$ Provided

$$r_{10} + r_{11}\frac{\mu}{m} > r_{22}\tilde{T} + A\frac{r_2(\tilde{T})}{\alpha_1 a(\tilde{T})}$$

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LINEAR STABILITY ANALYSIS

The local stability analysis of the equilibrium points can be studied from the variational matrix of the mathematical model given by (3.1) to (3.4) as follows:

$$V_1^T = \begin{bmatrix} -\frac{s_0\overline{H}}{k_1(T)} & -a(\overline{T})\overline{H} & r_{11}\overline{H} & -r_{22}\overline{H} - \frac{s_0\overline{H}^2}{k^2{}_1(T)}k_{11} \\ 0 & -r_2(\overline{T}) + \alpha_1 a(\overline{T})\overline{H} & 0 & 0 \\ r_2\overline{N} & 0 & -m\overline{N} & 0 \\ 0 & 0 & 0 & -\alpha \end{bmatrix}$$

The characteristics equation of V_1^T is given as

$$\left|V_1^T - \lambda I\right| = 0 .$$

After the simplification by mathematica-5 it was found that the equilibrium point E_1 is not locally stable as the condition $\frac{s_0 m}{k(\bar{T})} < -r_2\alpha_1$ is not possible. For the linear stability analysis of the equilibrium point $E_{2,}$ linearize the system (3.1) to (3.4) using the following transformations or perturbations-

$$H = \widetilde{H} + n_1, P = \tilde{P} + n_2,$$

$$N = \widetilde{N} + n_3, T = \tilde{T} + n_4 \quad .$$

Neglecting the higher powers and the products of the perturbations, the linearized system is given by-

$$\frac{dn_1}{dt} = -\frac{s_0\widetilde{H}}{k_1(\tilde{T})}n_1 - a(\tilde{T})\widetilde{H}n_2 + r_{11}\widetilde{H}n_3 - \left\{r_{22}\widetilde{H} + \frac{s_0\widetilde{H}^2}{K_1^2(\tilde{T})} - a_1\tilde{P}\widetilde{H}\right\}n_4 \tag{5.1}$$

$$\frac{dn_2}{dt} = \alpha_1 a(\tilde{T})\tilde{P}n_1 - \gamma\tilde{P}n_2 - \left(r_{22}\tilde{P} + \alpha_1 a_{11}\tilde{P}\widetilde{H}\right)n_4 \tag{5.2}$$

$$\frac{dn_3}{dt} = -\alpha_2\widetilde{N}n_1 - m\widetilde{N}n_3 \tag{5.3}$$

$$\frac{dn_4}{dt} = -\alpha n_4 \tag{5.4}$$

Consider the Liapunov function X as:

$X = \frac{1}{2}(n_1^2 + A_1 n_2^2 + A_2 n_3^2 + A_3 n_4^2)$ *where* $A_i > 0 \;\forall\; i = 1,2,3$ are arbitrary constants (5.5).

Differentiating (5.5) w. r.t. ***"t"*** and using (5.1) to (5.4) in $\frac{dX}{dt}$, we get-

$$\frac{dX}{dt} = n_1\left\{-\frac{s_0\widetilde{H}}{k_1(\tilde{T})}n_1 - a(\tilde{T})\widetilde{H}n_2 + r_{11}\widetilde{H}n_3 - \left(r_{22}\widetilde{H} + \frac{s_0\widetilde{H}^2}{K_1^2(\tilde{T})} - a_1\tilde{P}\widetilde{H}\right)n_4\right\} + A_1 n_2\left\{\alpha_1 a(\tilde{T})\tilde{P}n_1 - \gamma\tilde{P}n_2 - \left(r_{22}\tilde{P} + \alpha_1 a_{11}\tilde{P}\widetilde{H}\right)n_4\right\} + A_2 n_3\left(-\alpha_2\widetilde{N}n_1 - m\widetilde{N}n_3\right) + A_3 n_4(-\alpha n_4) \tag{5.6}$$

Now using the inequality $\boldsymbol{a^2 + b^2 \geq \pm 2ab}$ in the R.H.S. expression of $\boldsymbol{\frac{dX}{dt}}$ given above we get-

$$\frac{dX}{dt} \leq -(S_1 n_1^2 + S_2 n_2^2 + S_3 n_3^2 + S_4 n_4^2) \tag{5.7}$$

Here, $\boldsymbol{\frac{dX}{dt}}$ is negative definite only when, $S_i > 0, \forall i = 1,2,3,4$ .Therefore, we find that the equilibrium point E_2 is locally asymptotically stable under the conditions given below:

$$S_i > 0\,,\ \forall\ i = 1,2,3,4,5 \ . \tag{5.8}$$

Where,

$$S_1 = \left\{\frac{s_0\widetilde{H}}{k_1(\tilde{T})} + \frac{a_1\tilde{P}\widetilde{H}}{2} - \frac{a(\tilde{T})\widetilde{H}}{2} - \frac{r_1\widetilde{H}}{2} - \frac{r_{22}\widetilde{H}}{2} - \frac{s_0\widetilde{H}^2}{K_1^2(\tilde{T})}\frac{k_{11}}{2} - \frac{A_1\alpha_1 a(\tilde{T})\tilde{P}}{2} - \frac{A_2\alpha_2\widetilde{N}}{2}\right\}$$

$$S_2 = A_1\left(\gamma\tilde{P} - \frac{\alpha_1 a(\tilde{T})\tilde{P}}{2} - \frac{r_{22}\tilde{P} + \alpha_1 a_{11}\tilde{P}\widetilde{H}}{2}\right)$$

$$S_3 = m\widetilde{N}A_2 - \frac{r_{11}\widetilde{H}}{2} - \frac{A_2\alpha_2\widetilde{N}}{2} \tag{5.9}$$

Choose, $A_2 = \frac{r_{11}\widetilde{H}}{2m\widetilde{N} - \alpha_2\widetilde{N}}$, set the condition $2m\widetilde{N} > \alpha_2\widetilde{N}$

$$S_4 = A_3\alpha - \frac{a_1\tilde{P}\widetilde{H}}{2} - \frac{s_0\widetilde{H}^2}{K_1^2(\tilde{T})}\frac{k_{11}}{2} - \frac{r_{22}\tilde{P}A_1}{2} - \frac{A_1\alpha_1 a(\tilde{T})\tilde{P}}{2}$$

NON LINEAR STABILITY ANALYSIS

For non-linear stability analysis of equilibrium point

E_2 let us consider the following region Δ given by-

Lemma 6A: The system (3.1)-(3.4) is uniformly bounded in the region Δ.

$\Delta = \{(H, P, N, T): 0 < P \leq P^u, 0 < H^l \leq H, 0 < T \leq T^u, 0 < N \leq N^u\}$

Theorem 6B: If the following conditions are satisfied then the equilibrium point E_2 is nonlinearly asymptotically stable.

$$2\frac{s_0}{k_1(\tilde{T})} > 3\{(1-\alpha)a(\tilde{T})\}^2, \qquad \frac{m}{3}\frac{s_0}{k_1(\tilde{T})} > \left(\frac{r_1+\alpha_2}{2}\right)^2,$$

$$\frac{\alpha}{3}\frac{s_0}{k_1(\tilde{T})} > \left\{\frac{r_{22}+\frac{s_0 H^l k_{11}}{k_1(\tilde{T})} - a_1 P^u}{2}\right\}^2,$$

$$2\gamma\alpha > (r_{22} + \alpha_1 a_{11} H^l)^2 \tag{6.7}$$

Stability Analysis of the Model with Diffusion (Dispersal) and Advection:

E_1 is linearly asymptotically stable under the following conditions:

$$\frac{s_0\bar{H}}{k_1(\bar{T})} + D_1\frac{\pi^2}{a^2} > \frac{a(\bar{T})\bar{H}}{2} + \frac{r_{11}\bar{H}}{2} + \frac{r_{22}\bar{H}}{2} + \frac{\alpha_2\bar{N}}{2},$$

$$r_2(\bar{T}) + D_2\frac{\pi^2}{a^2} > \alpha_1 a(\bar{T})\bar{H} + \frac{a(\bar{T})\bar{H}}{2}$$

$$m\bar{N} + D_3\frac{\pi^2}{a^2} > \frac{r_{11}\bar{H}}{2} + \frac{\alpha_2\bar{N}}{2},\ \alpha + D_4\frac{\pi^2}{a^2} > \frac{r_{22}\bar{H}}{2}. \tag{7a}$$

If the above conditions are not satisfied then the equilibrium point E_1 is locally unstable.

Remark - By taking diffusion (dispersal) into consideration it can be seen that the unstable equilibrium point E_1 will tend to stable situation under some conditions.

E_2 is linearly asymptotically stable under the following conditions:

$$\frac{s_0\tilde{H}}{k_1(\tilde{T})} + \frac{a_1\tilde{P}\tilde{H}}{2} + \frac{D_1}{(H^u)^2}\frac{\pi^2}{a^2} > \frac{\alpha_1 a(\tilde{T})\tilde{P}}{2} + \frac{a(\tilde{T})\tilde{H}}{2} + \frac{r_{11}\tilde{H}}{2} + \frac{r_{22}\tilde{H}}{2} + \frac{\alpha_2\tilde{N}}{2}, \gamma\tilde{P} + \frac{D_2}{(P^u)^2}\frac{\pi^2}{a^2} > \frac{\alpha_1 a(\tilde{T})\tilde{P}}{2} + \frac{(r_{22}\tilde{P}+\alpha_1 a_{11}\tilde{P}\tilde{H})}{2}$$

$$, m\tilde{N} + \frac{D_3}{(N^u)^2}\frac{\pi^2}{a^2} > \frac{r_{11}\tilde{H}}{2} + \frac{\alpha_2\tilde{N}}{2} \quad \alpha + \frac{a_1\tilde{P}\tilde{H}}{2} + D_4\frac{\pi^2}{a^2} > \frac{r_{22}\tilde{H}}{2} + \frac{(r_{22}\tilde{P}+\alpha_1 a_{11}\tilde{P}\tilde{H})}{2} \tag{7b}$$

If the above conditions are not satisfied then the equilibrium point E_2 is unstable.

Non Linear Stability Analysis of Equilibrium Point E_2:

Theorem 8A: If the following inequalities hold:

$$\frac{s_0}{k_1(\tilde{T})} + D_1\frac{\pi^2}{a^2} + \frac{a_1 P^u}{2} > \frac{(1+\alpha_1)a(\tilde{T})}{2} + \frac{\alpha_2+r_{11}}{2} + \frac{1}{2}\left(r_{22} + \frac{s_0 H^l k_{11}}{k_1(\tilde{T})}\right) \tag{8.6}$$

$$\gamma + D_2\frac{\pi^2}{a^2} > \frac{(1+\alpha_1)a(\tilde{T})}{2} + \frac{1}{2}(r_{22} + \alpha_1 a_{11} H^l), \qquad m + D_3\frac{\pi^2}{a^2} > \frac{\alpha_2+r_{11}}{2}$$

Then the equilibrium point E_2 is nonlinearly asymptotically stable.

Appendix

Proof of the Lemma6A: From the system (3.1)-(3.4), we have

$\frac{d(P+N+T)}{dt} \le Q_0 - \alpha T - (r_{20} - \alpha_1 a_0 k_1)P - (\alpha_2 H^l - \mu)N$ therefore,

$\lim_{t\to\infty}(P+N+T) \le \frac{Q_0}{\theta}$ where, $\theta = min\{\alpha, (r_{20} - \alpha_1 a_0 k_1), (\alpha_2 H^l - \mu)\}$ assuming $r_{20} > \alpha_1 a_0 k_1$ and $H^l = \frac{Q_0 - \frac{s_0 k_1^2}{k_1 T^u}}{\emptyset}$ where, $\emptyset = max\{(r_{20}T^u + a_0 P^u - r_{10}), \alpha\}$ assuming $r_{20}T^u + a_0 P^u > r_{10}$

Hence, $P^u = N^u = T^u = \frac{Q_0}{\theta}$, and the upper bound of H is its carrying capacity. Hence the system (3.1)-(3.4) is bounded.

Proof of Theorem 6B: Using the transformations or the perturbations about the Equilibrium point E_2-

$H = \tilde{H} + n_1, P = \tilde{P} + n_2, N = \tilde{N} + n_3, T = \tilde{T} + n_4$ The system (3.1) to (3.4) becomes-

$$\frac{1}{\tilde{H}+n_1}\frac{dn_1}{dt} = -\frac{s_0}{k_1(\tilde{T})}n_1 - a(\tilde{T})n_2 + r_{11}n_3 - \left(r_{22} + \frac{s_0 H k_{11}}{k_1(\tilde{T})} - a_1 P\right)n_4 \tag{6.1}$$

$$\frac{1}{\tilde{P}+n_2}\frac{dn_2}{dt} = \alpha_1 a(\tilde{T})n_1 - \gamma n_2 - (r_{22} + \alpha_1 a_{11} H)n_4 \tag{6.2}$$

$$\frac{1}{\tilde{N}+n_3}\frac{dn_3}{dt} = -\alpha_2 n_1 - m n_3 \tag{6.3}$$

$$\frac{dn_4}{dt} = -\alpha n_4 \qquad (6.4)$$

Consider the positive definite function:

$$Y = \left\{n_1 - \tilde{H}log\left(1 + \frac{n_1}{\tilde{H}}\right)\right\} + \left\{n_2 - \tilde{P}log\left(1 + \frac{n_2}{\tilde{P}}\right)\right\} + \left\{n_3 - \tilde{N}log\left(1 + \frac{n_3}{\tilde{N}}\right)\right\} + \frac{1}{2}n_4^2 \qquad (6.5)$$

Differentiating (6.5) w.r.t. t and using the system of equations (6.1)-(6.4) and the region of attraction Δ in $\frac{dY}{dt}$ given above, we get-

$$\frac{dY}{dt} \leq -\left[\left\{\frac{s_0}{k_1(\tilde{T})}\frac{n_1^2}{3} + (1-\alpha)a(\tilde{T})n_1n_2 + \gamma\frac{n_2^2}{2}\right\} + \left\{\frac{s_0}{k_1(\tilde{T})}\frac{n_1^2}{3} + (r_1+\alpha_2)n_1n_3 + mn_3^2\right\} + \left\{\frac{s_0}{k_1(\tilde{T})}\frac{n_1^2}{3} + \left(r_{22} + \frac{s_0H^lk_{11}}{k_1(\tilde{T})} - a_1P^u\right)n_1n_4 + \alpha n_4^2\right\} + \left\{\gamma\frac{n_2^2}{2} + (r_{22} + \alpha_1a_{11}H^l)n_2n_4 + \alpha n_4^2\right\}\right] \qquad (6.6)$$

By using Sylverster's criterion we have the following conditions for $\frac{dY}{dt}$ to be negative definite if the conditions given in (6.7) are satisfied.

Hence, from Liapunov theorem we conclude that the equilibrium point E_2 is nonlinearly asymptotically stable under the conditions given by (6.7).

Proof of the theorem 8A:

Using the following transformations or the perturbations about the equilibrium point E_2-

$$H(x,t) = \tilde{H} + v_1(x,t), P(x,t) = \tilde{P} + v_2(x,t)$$

$$, N(x,t) = \tilde{N} + v_3(x,t), T(x,t) = \tilde{T} + v_4(x,t)$$

Then the system (2.1) to (2.4) reduces to:

$$\frac{1}{\tilde{H}+v_1}\frac{dn_1}{dt} = -\frac{s_0}{k_1(\tilde{T})}v_1 - a(\tilde{T})v_2 + r_{11}v_3 - \left\{r_{22} + \frac{s_0Hk_{11}}{k_1(\tilde{T})} - a_1P\right\}v_4 + \frac{D_1}{H}\frac{\partial^2 v_1}{\partial x^2} \qquad (8.1)$$

$$\frac{1}{\tilde{P}+v_2}\frac{dv_2}{dt} = \alpha_1a(\tilde{T})v_1 - \gamma v_2 - (r_{22} + \alpha_1a_{11}H)v_4 - \frac{u_2}{P}\frac{\partial v_2}{\partial x} + \frac{D_2}{P}\frac{\partial^2 v_2}{\partial x^2} \qquad (8.2)$$

$$\frac{1}{\tilde{N}+v_3}\frac{dv_3}{dt} = -\alpha_2v_1 - mv_3 - \frac{u_3}{N}\frac{\partial v_3}{\partial x} + \frac{D_3}{N}\frac{\partial^2 v_3}{\partial x^2} \qquad (8.3)$$

$$\frac{dv_4}{dt} = -\alpha v_4 + D_4\frac{\partial^2 v_4}{\partial x^2} \qquad (8.4)$$

Consider the positive definite function:

$$G(t) = \int_0^a \left[\left\{v_1 - \tilde{H}log\left(1 + \frac{v_1}{\tilde{H}}\right)\right\} + \left\{v_2 - \tilde{P}log\left(1 + \frac{v_2}{\tilde{P}}\right)\right\} + \left\{v_3 - \tilde{N}log\left(1 + \frac{v_3}{\tilde{N}}\right)\right\} + \frac{1}{2}v_4^2\right]dx$$

Differentiating $G(t)$ w.r.t. t, we have-

$$\frac{dG}{dt} = \int_0^a \left(\frac{v_1}{\tilde{H}+v_1}\frac{dn_1}{dt} + \frac{v_2}{\tilde{P}+v_2}\frac{dn_2}{dt} + \frac{v_3}{\tilde{N}+v_3}\frac{dn_3}{dt} + \frac{dv_4}{dt}\right)dx$$

Using the system from (8.1) to (8.4), the Poincare's inequality and $a^2 + b^2 \geq \pm 2ab$ in $\frac{dG}{dt}$,we get-

$$\frac{dG}{dt} \leq -\left[\int_0^a \left\{\frac{s_0}{k_1(\tilde{T})} + \frac{D_1}{(H^u)^2}\frac{\pi^2}{a^2} - \frac{(1+\alpha_1)a(\tilde{T})}{2} - \frac{\alpha_2 + r_{11}}{2} - \frac{1}{2}\left(r_{22} + \frac{s_0H^lk_{11}}{k_1(\tilde{T})} - a_1P^u\right)\right\}v_1^2 + \left\{\gamma + \frac{D_2}{(P^u)^2}\frac{\pi^2}{a^2} - \frac{(1+\alpha_1)a(\tilde{T})}{2} - \frac{1}{2}\left(r_{22} + \alpha_1a_{11}H^l\right)\right\}v_2^2 + \left\{m + \frac{D_3}{(N^u)^2}\frac{\pi^2}{a^2} - \frac{\alpha_2 + r_{11}}{2}\right\}v_3^2 + \left(\alpha + D_4\frac{\pi^2}{a^2}\right)v_4^2\right]dx \qquad \textbf{(8.5)}$$

Here we have assumed in the manuplation that:

$$0 < H^l \leq H \leq H^u, 0 < P^l \leq P \leq P^u, 0 < N^l \leq N \leq N^u, 0 < T^l \leq T \leq T^u$$

Therefore, $\frac{dG}{dt}$ is negative if all the coefficients of $v_1^2, v_2^2, v_3^2 and\ v_4^2$ are positive.Hence E_2 is nonlinearly asymptotically stable under the conditions (8.6).

Conclusion

In this paper, the stability analysis of two feasible equilibrium states E_1 and E_2 of the model has been carried out. From the linear stability analysis of the equilibrium point E_1 without diffusion and advection it has been found that the equilibrium point E_1 is linearly unstable but with diffusion and advection it is conditionally stable. From the stability of this equilibrium point we conclude that the predator population will die out and prey population will survive and further, from the equilibrium value, it may also be concluded that prey population level goes down because of the increase in temperature and the plant biomass decreases if prey population increases. The equilibrium point E_2 is linearly asymptotically stable with and without diffusion and advection. The equilibrium point

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E_2 is also non-linearly asymptotically stable under some conditions from which we conclude that the prey and predator population will co-exist but at lower equilibrium level due to the effect of increased temperature**.**

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